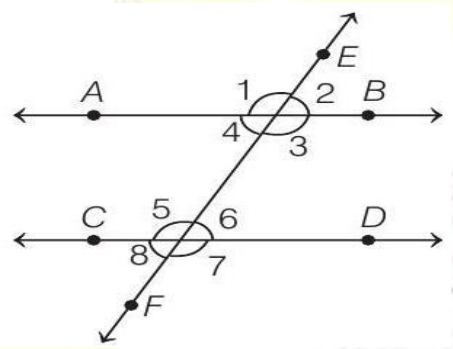
Geometry

Plane Geometry

Plane geometry is about flat shapes like line, circle and triangle etc. These types of shapes can be easily drawn on a piece of paper.

1. Lines and Angles: Two lines in the same plane are said to be parallel, if they never meet. A line which cuts a pair of parallel line is called a transversal.



Here, two parallel lines AB and CD are cut by a transversal i.e., EF. Then,

- The corresponding angles are $\angle 1 = \angle 5, \angle 2 = \angle 6, \angle 4 = \angle 8$ and $\angle 3 = \angle 7$.
- The alternate angles are $\angle 1 = \angle 7, \angle 2 = \angle 8, \angle 3 = \angle 5$ and $\angle 4 = \angle 6$.

2. Triangles: A figure bounded by three straight lines is called a triangle. The small of all interior angles of a triangle is $180°$.

Types of Triangles: **Equilateral Triangle A** triangle having all sides equal is called an equilateral triangle and each angle equal to $60°$.

Scalene Triangle A triangle having all sides of different length is called a scalene triangle.

Isosceles Triangle A triangle having two sides equal is called an isosceles triangle.

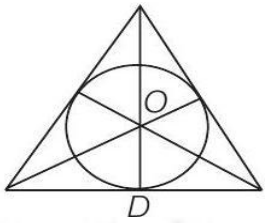
Right Angled Triangle A triangle one of whose angle measures $90°$ is called a right angled triangle.

Obtuse Angled Triangle A triangle one of whose angles lies between 90° and 180° is called an obtuse angled triangle.

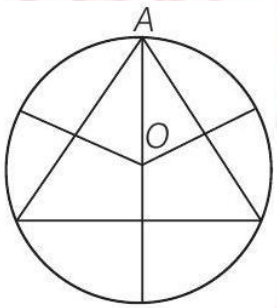
Acute Angled Triangle A triangle each of whose angle is less than 90° is called an acute angled triangle.

3. Circle: A circle is a set of points which are equidistant from a given point. The given point is known as the center of that circle.

$$\text{Inradius } (OD) = \frac{1}{3} \times \text{ Height } = \frac{\text{Side}}{2\sqrt{3}}$$



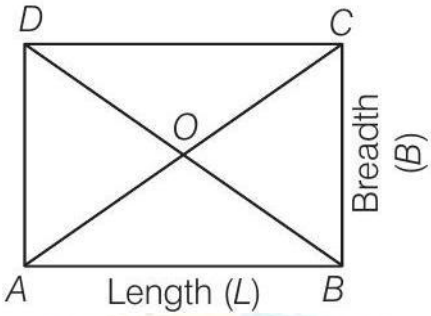
$$\text{Circumradius } (OA) = \frac{2}{3} \times \text{Height} = \frac{\text{Side}}{\sqrt{3}}$$



4. Quadrilateral: It is a plane figure bounded by four straight lines. The sum of the internal angles of a quadrilateral is equal to 360°.

Parallelogram

A quadrilateral in which the opposite sides are equal and parallel, is called a parallelogram.



- The opposite angles are equal in magnitudes.
- The diagonals of a parallelogram are not equal in magnitudes, but they bisect each other.
- $AC \neq BD$ but $AO = OC$ and $OB = OD$.

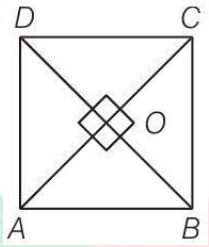
Rectangle

A parallelogram in which the adjacent sides are perpendicular to each other, is called a rectangle.

The diagonals of a rectangle are of equal magnitudes and bisect each other i.e., $AC = BD$ and $OA = OB = OC = OD$.

Square: A parallelogram in which all the sides are equal and perpendicular to each other, is called a square.

- The diagonals bisect each other at right angles and form four isosceles right angled triangles.



- The diagonals of a square are of equal mangnitudes i.e., $AC = BD$.

Polygon: Regular polygon A polygon in which all the sides are equal and also all the interior angles are equal, is called a regular polygon.

- Sum of all interior angles $= (n - 2) \times 180°$

$$= (2n - 4) \times 90°$$

- Each interior angle $= 180° -$ Exterior angle

- Each exterior angle $= \left(\frac{360^\circ}{\text{Number of sides}}\right)$ (in degrees)
- Sum of all exterior angle $= 360^\circ$ (always constant)
- Number of diagonals of polygon of n sides $= \frac{n(n-3)}{2}$

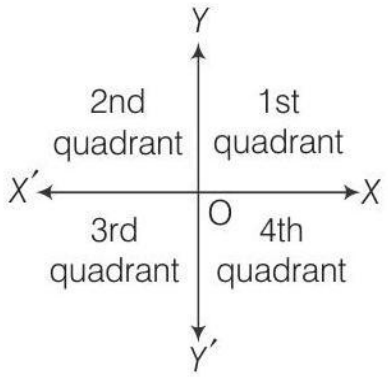
Coordinate Geometry

It is a system of geometry, where the position of points on the plane is described by using an ordered pair of numbers.

Quadrants

The X and Y-axes divide the cartesian plane into four regions referred as quadrants.

The table of sign conventions of coordinates in various quadrants is given below



ıdrɛgiœof (ımp
I ′OY-, +2,3)
II OX-, +2,4
III ′OY-, −1, −
IV ′OX-, −, −3

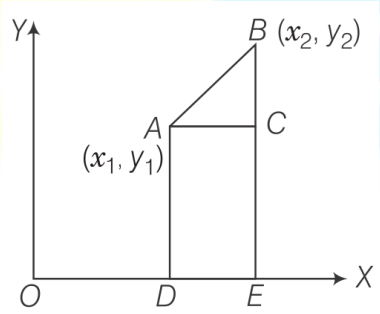
The coordinates of point O (origin) are taken as $(0,0)$.
The coordinates of any point on X-axis are of the form $(x, 0)$.

The coordinates of any point on Y-axis are of the form $(0, y)$.

Distance Formula

Distance between two points

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points, then



$$\boldsymbol{AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}$$

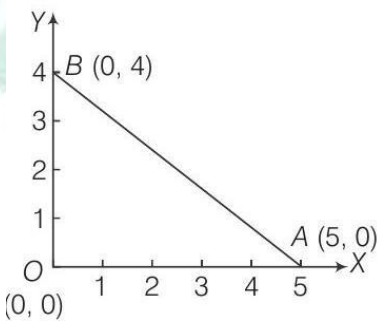
Example: The distance between $A(1,2)$ and $B(5,6)$ is

$$AB = \sqrt{(5-1)^2 + (6-2)^2} = \sqrt{4^2 + 4^2}$$
$$= \sqrt{32} = 4\sqrt{2} \text{ units}$$

Area of a Triangle

If $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are three vertices of a $\triangle ABC$, then its area is given by

Area of $\triangle ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$



Example: If we have to find the area of a triangle having the vertices $(0,0), (5,0)$ and $(0,4)$, then

$\therefore$ Area of triangle $= \frac{1}{2}\{0(0-4)+5(4-0)+0(0-0)\}$

$= \frac{1}{2} \times 20 = 10$ sq units

$\because (x_1, y_1) = (0,0), (x_2, y_2) = (5,0)$ and $(x_3, y_3) = (0,4)$

Collinearity of Three Points

Three points $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear, if

(i) Area of $\triangle ABC$ is 0 , i.e.

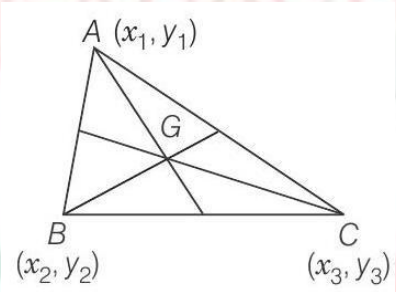
$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0.$

(ii) Slope of AB = Slope of BC = Slope of AC
(iii) Distance between A and B + Distance between B and C = Distance between A and C

Centroid of a Triangle

Centroid is the point of intersection of all the three medians of a triangle.

If $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of $\triangle ABC$, then the coordinates of its centroid are



$$\left[\frac{1}{3}(x_1 + x_2 + x_3), \frac{1}{3}(y_1 + y_2 + y_3)\right]$$

Section formulae

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points on the cartesian plane. Let point $P(x, y)$ divides the line AB in the ratio of m: n internally.

Then,

$$x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n}$$

If P divides AB externally, then

$$x = \frac{mx_2 - nx_1}{m - n}, y = \frac{my_2 - ny_1}{m - n}$$

If P is the mid-point of AB, then

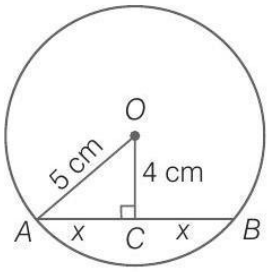
$$x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$$

Solved Examples:

1. A chord AB is drawn in a circle with centre O and radius 5 cm. If the shortest distance between centre and chord is 4 cm, find the length of chord AB?

(a) 6 cm
(b) 5 cm
(c) 4 cm
(d) 3 cm

Sol. (a) In the adjoining figure $AO = 5$ cm (radius)



$OC = 4$ cm (shortest distance between centre and chord)

Let length of chord AB be $2x$, then $AC = x$.

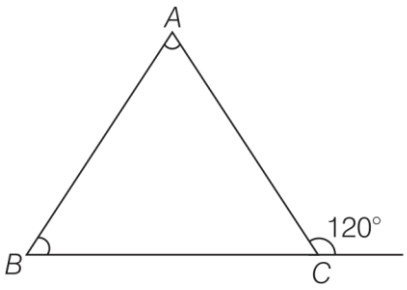
In $\triangle AOB$, $AO^2 = AC^2 + OC^2$

$\Rightarrow (5)^2 = x^2 + (4)^2 \Rightarrow 25 = x^2 + 16$

$\therefore x = \sqrt{25 - 16} = \sqrt{9} = 3$ cm

$\therefore$ Length of chord $AB = 2x = 2 \times 3 = 6$ cm

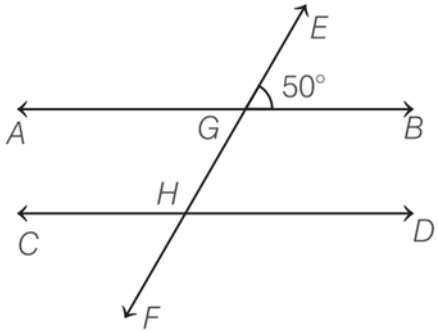
2. In the figure given, $\angle BAC : \angle ABC = 2:3$. Find the measure of $\angle ABC$.



(a) Obtuse angle
(b) Acute angle
(c) Right angle
(d) None of these

Sol. (b) Let $\angle A = 2x$ and $\angle B = 3x$
Then, $2x + 3x = 120°$
[Exterior angle is equal to the sum of the interior opposite
$\Rightarrow\ 5x = 120°$
$\Rightarrow\ x = 24°$
$\therefore\ \angle ABC = 3x = 3 \times 24° = 72°$

3. In the given figure, AB and CD are parallel lines.
If $\angle EGB = 50°$, find $\angle CHG$.



(a) 120°
(b) 130°
(c) 125°
(d) 140°

Sol. (b) $\angle AGH = \angle EGB$ [Vertically opposite angles]

$\angle AGH = 50°$

Now, $\angle AGH + \angle CHG = 180°$

[Interior angles on the same side of the transversal are supplementary.]

$\therefore 50° + \angle CHG = 180° \Rightarrow \angle CHG = 180° - 50° = 130°.$

4. An angle $\theta°$ is one-fourth of its supplementary angle. What is the measure of the angle $\theta°$?

(a) 36°
(b) 34°
(c) 32°
(d) 31°

Sol. (a) If the sum of two angles is 180°, the angles are said to be supplementary.

$\therefore$ The supplementary angle of $\theta°$ is $(180° - \theta°)$.

Given that, $\theta° = \frac{1}{4}(180° - \theta°)$

$\Rightarrow 4\theta° = 180° - \theta° \Rightarrow 5\theta° = 180°$

$\Rightarrow \theta° = \frac{180°}{5} = 36°$

5. The number of diagonals in a 27-sided polygon is

(a) 324
(b) 325
(c) 322
(d) 320

Sol. (a) Number of diagonals of polygon of n sides $= \frac{(n)(n-3)}{2}$

Number of diagonals of polygon of 27 sides $= \frac{27\times24}{2} = 324$

6. Find the distance between the points $A(-6,8)$ and $B(4,-8)$.

(a) 18.86 units
(b) 16.76 units
(c) 11.77 units
(d) 16.76 units

Sol. (a) Here, $A(-6,8) = A(x_1, y_1)$ and $B(4,-8) = B(x_2, y_2)$

So, $x_1 = -6, y_1 = 8,\ x_2 = 4$ and $y_2 = -8$

∴ Required distance,

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{\{4 - (-6)\}^2 + (-8 - 8)^2}$$
$$= \sqrt{(4 + 6)^2 + (-16)^2} = \sqrt{100 + 256}$$
$$= \sqrt{356} \simeq 18.86 \text{ units}$$

7. Find the area of $\triangle ABC$, whose vertices are $A(8,-4), B(3,6)$ and $C(-2,4)$.

(a) 20 sq units
(b) 30 sq units
(c) 35 sq units
(d) 29 sq units

Sol. (b) Here, $A(8,-4)$, then $x_1 = 8, y_1 = -4$

$B(3,6)$, then $x_2 = 3, y_2 = 6$
$C(-2,4)$, then $x_3 = -2, y_3 = 4$

∴ Area of $\triangle ABC$

$$= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$
$$= \frac{1}{2}\{8(6 - 4) + 3(4 - (-4)) + (-2)(-4 - 6)\}$$
$$= \frac{1}{2}\{16 + 24 + 20\} = \frac{1}{2} \times 60 = 30 \text{ sq units}$$