Average

An average or arithmetic mean of given data is the sum of the given observations divided by number of observation.

Average

The average of a number of quantities belongs to some kind is their sum divided by the number of those quantities.

In other words, an average of a given observation or data is a number which is found on dividing the sum of observations or data by the number of observations or data given.

$$\textbf{Average} = \frac{\text{Sum of observations}}{\text{Number of observations}}$$

Points to be Remember

- The quantities whose average is to be determined, should be in the same unit.
- Average of the given observations is x.

  (i) If each observation is increased by a, then new average $= x + a$

  (ii) If each observation is decreased by a, then new average $= x - a$

  (iii) If each observation is multiplied by a, then new average $= ax$

  (iv) If each observation is divided by a, then new average $= \frac{x}{a}$

Important Tips/Formulae

- Average of two groups are a and b, respectively. If number of items in first and second group are x and y respectively, then Average of both the groups $= \frac{ax+by}{x+y}$
- The average weight/age of group of n person is x. If a person of y kg/yr is replaced by another person of z kg/yr, then New average weight/age $= x + \left(\frac{z-y}{n}\right)$

- The average of marks of n students was calculated as x. But it was later found that the marks of one student had been wrongly entered as a instead of b and of another as c instead of d.
- The correct average

$$= x + \left[\frac{\text{Sum of correct marks } - \text{Sum of wrongly entered marks}}{n}\right]$$

- The average of consecutive natural numbers upto $n = \frac{n+1}{2}$
- The average of square of consecutive natural numbers upto n

$$= \frac{(n+1)(2n+1)}{6}$$

- The average of cubes of consecutive natural numbers upto n

$$= \left[n\left(\frac{n+1}{2}\right)\right]^2$$

- The average of consecutive even numbers upto $n = \frac{n}{2} + 1$
- The average of consecutive odd numbers upto $n = \frac{n+1}{2}$
- The average of first n consecutive even numbers $= (n+1)$
- The average of first n consecutive odd numbers $= n$

- A man covers a distance d at x km/h and returns back to the starting point at y km/h. Then,

Average speed during whole journey $= \frac{2xy}{(x+y)}$ km/h

Solved Examples:

1. If average of given numbers $3,5,6,7,9,x$ is 7, then find the value of x.

(a) 12
(b) 10
(c) 14
(d) 11

Sol. (a) Average $= \frac{3+5+6+7+9+x}{6}$

i.e., $7 = \frac{30 + x}{6}$

$\Rightarrow 30 + x = 42 \Rightarrow x = 12$

2. A tabulator while calculating the average marks of 100 students of an examination, by mistake enters 68, instead of 86 and obtained the average as 58, the actual average marks of those students is

(a) 58.18
(b) 57.82
(c) 58.81
(d) 57.28

Sol. (a) Actual total marks of 100 students

$$= 5800 + (86 - 68) = 5818$$

$\therefore$ Required actual average $= \frac{5818}{100} = 58.18$

3. The average weight of 8 persons increased by 1.5 kg when a person weighing 65 kg is replaced by a new person. What could be the weight of new person?

(a) 75 kg
(b) 82 kg
(c) 77 kg
(d) 70 kg

Sol. (c) Let the average weight of 8 persons be $\bar{x}$ and the weight of the new person be w kg.

Sum of the weight of 8 persons $= 8\bar{x}$

By given condition, new sum $= 8\bar{x} - 65 + w$

$\therefore$ New average $= \frac{8\bar{x} - 65 + w}{8} = \bar{x} + 1.5 \Rightarrow 8\bar{x} - 65 + w = 8\bar{x} + 12 \Rightarrow w = 77$ kg

Alternate Method

Weight of new person = number of persons × increment of average + weight of the person replaced

$= 8 \times 1.5 + 65 = 77$ kg

4. The average of 9 numbers is 30. The average of first 5 numbers is 25 and that of the last 3 numbers is 35. What is the 6th number?

(a) 42
(b) 40
(c) 39
(d) 41

Sol. (b) Average $= \dfrac{\text{Sum of observations}}{\text{Number of observations}}$

$\Rightarrow$ Total sum $= 9 \times 30 = 270$

Sum of first five $= 5 \times 25 = 125$

Sum of last three $= 3 \times 35 = 105$

$\therefore$ 6th number $= 270 - (125 + 105) = 40$

5. A library has an average number of 510 visitors on Sunday and 240 on other days. The average number of visitors per day in a month of 30 days beginning with Sunday is

(a) 290
(b) 285
(c) 295
(d) 300

Sol. (b) A month beginning with Sunday will have $= 5$

Sundays total number of other days $= 30 - 5 = 25$ days

$\therefore$ Average number of visitor $= \dfrac{510 \times 5 + 25 \times 240}{30}$

$= \dfrac{2250 + 6000}{30} = 285$ visitors