Square Root and Cube Root

"Square and Cubes are the basic operations with equal importance as the binary operations. To cater such problem you must have the conceptual clarity regarding the same".

Square

The square of any number is the number multiplied by itself.

e.g., $2^2, 3^2, 4^2, \ldots, n^2$

$$2^2 = 2 \times 2 = 4$$
$$4^2 = 4 \times 4 = 16$$
$$n^2 = n \times n$$

In $2^2, 3^2, 4^2, \ldots, n^2$, figure $2, 3, 4, \ldots, n$ are called the base and at the top of a number tells you to square it. The above statements can be expressed by saying that the square of 2 is 4 or two squared is 4 and so on.

Properties of Square

- It cannot be a negative number.

- It cannot have odd number of zeros at its end.

- It cannot end with 2,3,7 or 8 .

Perfect Square

A natural number is called a perfect square or a square number. If it is the square of some natural number. $n = m^2$, for some natural number m, then n is said to be a perfect square.

e.g., 4 is a perfect square of 2.9 is a perfect square of 3 . Note

(i) Squares of even numbers are always even.

(ii) Squares of odd numbers are a/ways odd.

Square Root

The square root of a number is that factor of the number which, when multiplied by itself, will give that number.

The above statement can be expressed by if a is the square root of ' b ', then ' b ' is the square of ' a '.

The square root of a number is indicated by the sign $\sqrt{}$ or $\sqrt[2]{}$.
The square root of 49 is written as $\sqrt{49}$.

Thus, $\sqrt{49} = \sqrt{7 \times 7} = 7$

Square root of a number can be learnt with the help of given below.

$\sqrt{81} = 9 \; \sqrt{100} = 10 \; \sqrt{121} = 11$
$\sqrt{144} = 12 \; \sqrt{169} = 13 \; \sqrt{196} = 14$
$\sqrt{225} = 15 \; \sqrt{256} = 16 \; \sqrt{289} = 17$
$\sqrt{324} = 18 \; \sqrt{361} = 19 \; \sqrt{400} = 20$
$\sqrt{441} = 21 \; \sqrt{484} = 22 \; \sqrt{529} = 23$
$\sqrt{576} = 24 \; \sqrt{525} = 25 \; \sqrt{676} = 26$
$\sqrt{729} = 27 \; \sqrt{784} = 28 \; \sqrt{841} = 29$

$\sqrt{900} = 30$ and so on ....

Unit Digit in Square

Number	Unit Digit	Number	Unit Digit
$(\ldots 1)^2$	... 1	$(\ldots 6)^2$	... 6
$(\ldots 2)^2$	... 4	$(\ldots 7)^2$	... 9
$(\ldots 3)^2$	... 9	$(\ldots 8)^2$	... 4
$(\ldots 4)^2$	... 6	$(\ldots 9)^2$	... 1
$(\ldots 5)^2$	... 5	$(\ldots 0)^2$	... 0

How to Calculate the Square Root?

There are two methods to calculate the square root.

Prime Factorisation Method

This method has the following steps.

Step I Express the given number as the product of prime factors

Step II Keep these factors in pairs.

Step III Take the product of these prime factors taking one out of every pair of the same primes. This product gives us the square root of the given number.
e.g. Find the square root of 1089 .

Sol. Prime factors of $1089 = 11 \times 11 \times 3 \times 3$

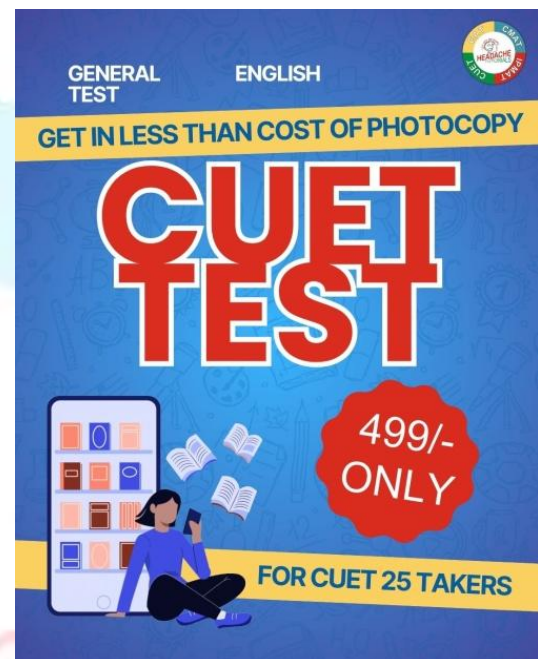| 11	1089
11	99
3	9
3	3
	1

$$\Rightarrow \sqrt{1089} = \sqrt{11 \times 11 \times 3 \times 3}$$

Now, taking one number from each pair and multiplying them, we get

$$\sqrt{1089} = 11 \times 3 = 33$$

e.g. Find the square root of 1024.

Sol. Prime factors of 1024

$$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$
$$\Rightarrow \sqrt{1024} = \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$$

Now, taking one number from each pair and multiplying them, we get $\sqrt{1024} = 2 \times 2 \times 2 \times 2 \times 2 = 32$

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2

Division Method

The steps of this method can be easily understood with the help of following examples

e.g. Find the square root of 18769.

Sol. Step I In the given number, mark off the digits in pairs starting from the unit digit. Each pair and the remaining one digit (if any), is called a period.

Step II Now, on subtracting, we get 0 (zero) as remainder.

Step III Bring down the next period, i.e., 87. Now the trial divisor is $1 \times 2 = 2$ and trial dividend is 87.

So, we take 23 as divisor and put 3 as quotient. The remainder is 18 now.

	137
1	1 87 69 1
23	87 69
267	1869 1869
	×

Step IV Bring down the next period, which is 69. Now, trial divisor is $13 \times 2 = 26$ and trial dividend is 1869. So, we take 267 as dividend and 7 as quotient. The remainder is 0 now.

Step V The process (processes like III and IV) goes on till all the periods (pairs) come to an end and we get remainder as 0 (zero). Hence, the required square root $= 137$

e.g. What is the square root of 151321?

Sol.

	389
3	15 13 21 9
68	613 544
769	6921 6921
	x

∴ Required square root $= 389$

Note Division method should be applied, when the given number is so large that it is very difficult to find its square root by the prime factorisation method.

Square of Two Digits

Let AB is 2 Digit number, then

Step I B^2

Step II $2(A \times B)$

Step III $A^2 = A^2 + 2AB + B^2$

e.g. $(16)^2$

Sol. Here, $1 = A$ and $6 = B$. Then,

$$\begin{aligned}
\text{Step I} \quad & (6)^2 = 36 \\
\text{Step II} \quad & 2(1 \times 6) = 12 \\
\text{Step III} \quad & (1)^2 = 1 = 1 + 12 + 36 = 2 + {}_1 5 + {}_3 6 = 256
\end{aligned}$$

e.g. $(23)^2$

Sol. Here, $2 = A$ and $3 = B$. Then,

$$\begin{aligned}
\text{Step I} \quad & (3)^2 = 9 \\
\text{Step II} \quad & 2(2 \times 3) = 12 \\
\text{Step III} \quad & (2)^2 = 4 = 4 + 12 + 9 = 5 + {}_1 2 + 9 = 529
\end{aligned}$$

e.g. $(26)^2$

Sol. Here, $2 = A$ and $6 = B$. Then,

$$\begin{aligned}
\text{Step I} \quad & (6)^2 = 36 \\
\text{Step II} \quad & 2(2 \times 6) = 24 \\
\text{Step III} \quad & (2)^2 = 4 = 4 + 24 + 36 = 6 + {}_2 7 + {}_3 6 = 676
\end{aligned}$$

e.g. $(47)^2$

Sol. Here, $4 = A$ and $7 = B$. Then,

$$\begin{aligned}
\text{Step I} \quad & (7)^2 = 49 \\
\text{Step II} \quad & 2(4 \times 7) = 56 \\
\text{Step III} \quad & (4)^2 = 16 = 16 + 56 + 49 \\
& = 22 + {}_6 0 + {}_4 9 = 2209
\end{aligned}$$

Square of a Number Ending in 5

Step I Multiply the number formed after deleting 5 at the unit's place with the number one higher than it.
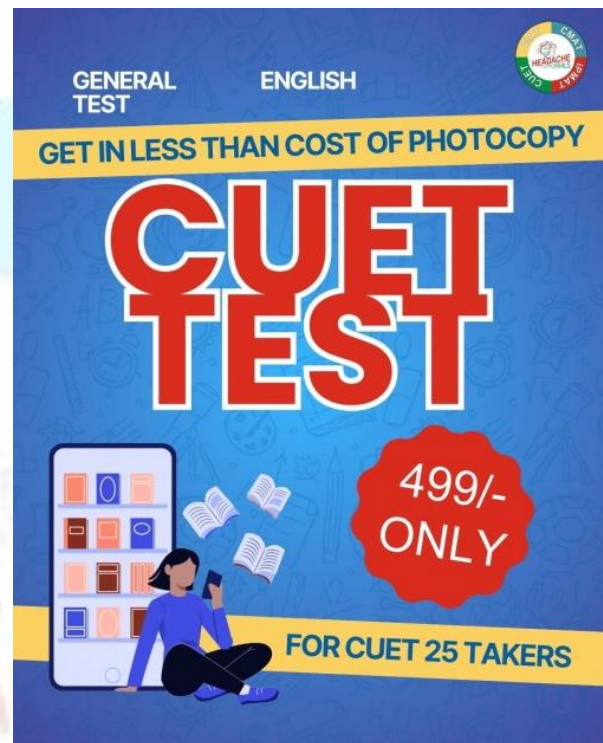
Step II Annex 25 on the right side of the product and you will get the square of the given number.

e.g. $(15)^2$

Sol. Step I $1 \times 2 = 2$
Step II $= 25 \Rightarrow (15)^2 = 225$
e.g. $(35)^2$

Square of Decimal Numbers

Step I Find the square of the number ignoring the decimal point.

Step II Put the decimal point in such a way that the number of decimal places in the square is twice of that in the original number.

Step III The square of a decimal number will lie between the square of its integral part and the square of the number one higher than the integral part.

e.g. $(3.6)^2 = (36)^2 = 1296 = 12.96$

e.g. $(4.5)^2 = (45)^2 = 2025 = 20.25$

e.g. $(10.5)^2 = (105)^2 = 11025 = 110.25$

Square of a Number Ending in 25

Step I Multiply the number formed after leaving 25 by a number 5 suffixed to it.

Step II Annex 625 to the right side of the product.

e.g. $(125)^2 = 1 \times 15 = 15$ hence, $(125)^2 = 15625$

e.g. $(225)^2 = 2 \times 25 = 50$ hence, $(225)^2 = 50625$

e.g. $(325)^2 = 3 \times 35 = 105$ hence, $(325)^2 = 105625$

How to Calculate the Square Root of Decimal Numbers?

If in a given decimal number, the number of digits after decimal are not even, then we put a 0 (zero) at the extreme right. Now, we mark off the periods and try to calculate the square root applying the division method.

1	147 · 1369 1
22	47 44
241	313 241
242 3	7269 7269

e.g. Find the square root of 147.1369.

Sol. 12.13

∴ Required square root = 12.13

How to Calculate the Square Root of a Fraction?

To find square root of a fraction, we have to find the square roots of numerators and denominators, separately.

e.g. $\sqrt{\dfrac{2704}{81}}$ is equal to

Sol. $\sqrt{\dfrac{2704}{81}} = \dfrac{\sqrt{2704}}{\sqrt{81}} = \dfrac{52}{9}$

$$
\begin{array}{r|l}
5 & 2704 \\
102 & 204 \\
& 204 \\
\hline
\end{array}
$$

Note Sometimes, numerator and denominator are not a complete square. In these types of cases, it is better to convert the given fraction into decimal fraction to find the square root

Cube

The cube of any number is the number multiplied by itself and by itself again.

e.g. $2^3, 5^3, \ldots, n^3$

A small figure 3 at the top of given number tells you to cube it. 2^3 is usually read as 'two cubed' or the 'cube of 2' and so on.

The cube of 5 is $5 \times 5 \times 5 = 125$

Perfect Cube

A natural number is said to be a perfect cube, if it is the cube of some natural number. A natural number n is a perfect cube, if $n = m^3$, where m is a natural number.

Numbers such as 1,8,27,64, ... are called perfect cube.

Note Cube of a number is the triple product obtained on multiplying the number by itself.

e.g. $9^3 = 9 \times 9 \times 9 = 729$

$11^3 = 11 \times 11 \times 11 = 1331$

$12^3 = 12 \times 12 \times 12 = 1728$ and so on....

Cube of a number can be learnt through this below

$$0^3 = 0 \qquad 1^3 = 1 \qquad 2^3 = 8$$
$$3^3 = 27 \qquad 4^3 = 64 \qquad 5^3 = 125$$
$$6^3 = 216 \qquad 7^3 = 343 \qquad 8^3 = 512$$
$$9^3 = 729 \qquad 10^3 = 1000 \qquad 11^3 = 1331$$
$$12^3 = 1728 \qquad 13^3 = 2197 \qquad 14^3 = 2744$$
$$15^3 = 3375 \qquad 16^3 = 4096 \qquad 17^3 = 4913$$
$$18^3 = 5832 \qquad 19^3 = 6859 \qquad 20^3 = 8000$$
$$21^3 = 92621 \qquad 22^3 = 10648 \qquad 23^3 = 12167$$
$$24^3 = 13824 \qquad 25^3 = 15625$$

Cube Root

The cube root of a number multiplied by itself and by itself again gives the number.

A natural number m is the cube root of a number n if $n = m^3$.

The above statement can be expressed by if m is the cube root of n, then n is the cube of m.e.g., 125 is the cube of 5 and therefore 5 is the cube root of 125.

How to Calculate the Cube Root?

Prime Factorisation Method

This method has following steps

Step I Express the given number as the product of prime factors.

Step II Keep these factors in a group of three.

Step III Take the product of these prime factors picking one out of every group (group of three) of the same primes. This product gives us the cube root of given number.

e.g., Find the cube root of 9261.

Sol. Prime factors of $9261 = (3 \times 3 \times 3) \times (7 \times 7 \times 7)$

$$\sqrt[3]{9261} = \sqrt[3]{3 \times 3 \times 3 \times 7 \times 7 \times 7}$$

Now, taking one number from each group of these,

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

we get $\sqrt[3]{9261} = 3 \times 7 = 21$

Important Tips/Formulae

- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

- $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$

- The value of $\sqrt{x \pm \sqrt{x \pm \sqrt{x \pm \cdots \infty}}}$

Suppose ' a ' and ' b ' are consecutive factors of x where $b > a$. If their sign is (+) in the expression, the answer is b i.e., bigger factor and if their sign is (-), the answer is a i.e., the smaller factor.

e.g., $\sqrt{20 - \sqrt{20 - \sqrt{20 - \sqrt{20 \ldots \infty}}}}$ is equal to

Sol. $20 = 4 \times 5$. Since, the sign is $(-)$, then the required answer is the smaller factor i.e., 4.

- $\sqrt{x \cdot \sqrt{x \cdot \sqrt{x \cdot \sqrt{x \ldots \infty}}}}$ If the root goes upto ∞ in multiplication, the answer is x itself.

Solved Examples:

1. Find the value of $\dfrac{\sqrt{288}}{\sqrt{128}}$.

   (a) $\dfrac{2}{3}$

   (b) $\dfrac{3}{2}$

   (c) $\dfrac{1}{2}$

   (d) 2

Sol. (b) $\dfrac{\sqrt{288}}{\sqrt{128}} = \sqrt{\dfrac{288}{128}} = \sqrt{\dfrac{144}{64}} = \dfrac{12}{8} = \dfrac{3}{2}$

2. If $\sqrt{1369} = 37$; find the value of

$$\sqrt{13.69} + \sqrt{0.1369} + \sqrt{0.001369} + \sqrt{0.00001369}$$

(a) 4.1207
(b) 4.1109
(c) 4.1107
(d) 3.8506

Sol. (c) We have,

Given expression $= \sqrt{\dfrac{1369}{100}} + \sqrt{\dfrac{1369}{10000}}$

$$+ \sqrt{\dfrac{1369}{1000000}} + \sqrt{\dfrac{1369}{100000000}}$$

$$= \dfrac{37}{10} + \dfrac{37}{100} + \dfrac{37}{1000} + \dfrac{37}{10000}$$

$$= 3.7 + 0.37 + 0.037 + 0.0037 = 4.1107$$

3. Find the smallest number which should be added to 8958 so that the sum is a perfect square.

   (a) 65
   (b) 63

(c) 69
(d) 67

Sol. (d)

$$
\begin{array}{c|c|c}
9 & 8958 & 94 \\
\dfrac{9}{184} & \underline{81} & 858 \\
\dfrac{4}{188} & \dfrac{736}{122} & \\
\end{array}
$$

$\therefore$ Required number $= (95)^2 - 8958 = 9025 - 8958 = 67$

4. Find $\sqrt[3]{1 - \dfrac{91}{216}}$.

   (a) $\dfrac{4}{5}$

   (b) $\dfrac{5}{6}$

   (c) $\dfrac{7}{8}$

   (d) $\dfrac{3}{4}$

Sol. (b) $\sqrt[3]{\dfrac{216 - 91}{216}} = \sqrt[3]{\dfrac{125}{216}}$

$$
= \sqrt[3]{\dfrac{5 \times 5 \times 5}{6 \times 6 \times 6}} = \dfrac{5}{6}
$$

5. Evaluate $\sqrt[3]{0.064} + \sqrt{1.21}$.

   (a) 1.8
   (b) 1.3
   (c) 1.5
   (d) 2.2

Sol. (c) $\sqrt[3]{0.064} + \sqrt{1.21} = \sqrt[3]{\dfrac{64}{1000}} + \sqrt{\dfrac{121}{100}}$

$$= \frac{\sqrt[3]{64}}{\sqrt[3]{1000}} + \frac{\sqrt{121}}{\sqrt{100}}$$

$$= \frac{4}{10} + \frac{11}{10} = 0.4 + 1.1 = 1.5$$

6. Find the value of n, if $\sqrt{3^n} = 729$

    (a) 11
    (b) 10
    (c) 12
    (d) 13

Sol. (c) $\sqrt{3^n} = 729$

$\sqrt{3^n} = 3^6$

$\therefore \ 3^n = (3^6)^2 = 3^{12}$

$\Rightarrow \ n = 12$