Ratio and Proportion

The number of times one quantity contains another quantity of the same kind is called ratio of the two unit.

Ratio

It is a tool to compare two or more numbers of same quantities.

Or

The ratio of two quantities in the same units is the fraction that one quantity is of the other.

Thus, the ratio a to b is the fraction $\frac{a}{b}$ written as $a:b$.

Note In the ratio $a:b$, the first term a is antecedent and second term b is consequent.

Properties of Ratios

(i) The value of a ratio remains unchanged, if each one of its terms is multiplied or divided by a same non-zero number.

(ii) $a^2:b^2$ is the duplicate ratio of $a:b$.

(iii) $a^3:b^3$ is the triplicate ratio of $a:b$.

(iv) $\sqrt{a}:\sqrt{b}$ is the sub-duplicate ratio of $a:b$.

(v) If $a:b$ and $c:d$ are two ratios, then the compounded ratio is $ac:bd$.

Proportion

The equality of two ratios is called proportion. Let a, b, c and d are four quantities, then the proportional are $a:b::c:d$.

Note In the proportion $a:b::c:d$, a and d are extreme values and b and c are mean values.

i.e., Product of means = Product of extreme

Properties of Proportion

(i) If x is the third proportional to a, b, then $a : b :: b : x$.

(ii) Mean proportional between a and b is $\sqrt{ab}$.

(iii) If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

and $\frac{a-b}{a+b} = \frac{c-d}{c+d}$ (Componendo and Dividendo)

e.g., Divide ₹ 1024 among, A, B and C so find the value of A and B.

Sol. Sum of the ratios = 4 + 5 + 7 = 16

$$\therefore \text{ First part } = \left(1024 \times \frac{9}{16}\right) = ₹576$$

$$\text{and second part } = \left(1024 \times \frac{7}{16}\right) = ₹448$$

Important Tips/Formulae

– If $\frac{a}{b} = 1, \frac{a+x}{a+x} = 1$ and $\frac{a-x}{a-x} = 1$

– If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots = k$

Then, $\frac{a+c+e+\cdots}{b+d+f+\cdots} = k$

– If $A : B$ and $B : C$ are given, then $A : B : C$ is given by $A : B$

$$\begin{tikzcd} A \arrow[d] & : & B \arrow[d] \\ B \arrow[urr] & : & C \end{tikzcd}$$

i.e., $A \times B : B \times B : B \times C$

e.g., If A : B = 5 : 4

and B : C = 3 : 2, then A:B:C = ?

Sol.

5 : 4

3 : 2

3 : 2

= 5 × 3 : 3 × 4 : 4 × 2

= 15 : 12 : 8

• If $\frac{a}{b}=\frac{N_1}{D_1},\frac{b}{c}=\frac{N_2}{D_2},\frac{c}{d}=\frac{N_3}{D_3}$, then $a:b:c:d$ is given by

$N_1N_2N_3 : D_1N_2N_3 : D_1D_2D_3$

Solved Examples:

1. If $\frac{a}{b}=\frac{3}{4},\frac{b}{c}=\frac{5}{6},\frac{c}{d}=\frac{7}{8}$, then find $a:b:c:d$.

(a) $105:120:135:180$
(b) $105:140:168:192$
(c) $105:140:158:212$
(d) $115:140:168:182$

Sol. (b) $a:b:c:d$

$= N_1N_2N_3 : D_1N_2N_3 : D_1D_2N_3 : D_1D_2D_3$

$= 3\times5\times7 : 4\times5\times7 : 4\times6\times7 : 4\times6\times8 = 105:140:168:192$

2. What is the ratio of 80 paise and ₹5?

(a) $2:25$

(b) $4:35$
(c) $4:25$
(d) $6:35$

Sol. (c) ₹ $5 = 5 \times 100$ paise

Their ratio $= \frac{80}{500} = 4:25$

3. Two natural numbers are in the ratio $3:5$ and their product is 2160. The smaller of the numbers is

(a) 36
(b) 24
(c) 18
(d) 12

Sol. (a) Let the natural numbers are $3x$ and $5x$, then

$$3x \times 5x = 2160 \Rightarrow 15x^2 = 2160$$
$$\Rightarrow \quad x^2 = 144 \Rightarrow x = 12$$

Hence, smaller $= 3x = 3 \times 12 = 36$

4. What must be added to each term of the ratio $7:11$ so as to make it equal to $3:4$?

(a) 8
(b) 7.5
(c) 6.5
(d) 5

Sol. (d) Let the required number is x, then $\frac{7+x}{11+x} = \frac{3}{4} \Rightarrow 28 + 4x = 33 + 3x \Rightarrow 4x - 3x = 33 - 28 \Rightarrow x = 5$

5. If $x:y = 3:1$, then find the ratio of $x^3 - y^3 : x^3 + y^3$.

(a) $12:11$
(b) $13:21$
(c) $13:14$
(d) $21:42$

Sol. (c) Let $x = 3k; y = k$

Then, $x^3 - y^3 = 27k^3 - k^3 = 26k^3$

and $\quad x^3 + y^3 = 27k^3 + k^3 = 28k^3$

Their ratio $= \frac{26k^3}{28k^3} = 13:14$

6. If two numbers are in the ratio of $5:8$ and if 9 be added to each, the ratio becomes $8:11$. Find the lower number.

(a) 10
(b) 13
(c) 12
(d) 15

Sol. (d) Let the numbers are $5x$ and $8x$.

By given condition, $\frac{5x+9}{8x+9} = \frac{8}{11}$

$\Rightarrow \quad 64x + 72 = 55x + 99$
$\Rightarrow \quad 9x = 27$
$\Rightarrow \quad x = 3$

Hence, lower number $= 5 \times 3 = 15$